

# 國立高雄師範大學九十五學年度博士班招生考試試題

系所別：科學教育研究所

科目：分析與代數（全一頁）

一、(1)  $(X, S, \mu)$  is a finite measure space

(2)  $1 \leq p < q \leq \infty$

Prove that  $L^q(X, S, \mu) \subseteq L^p(X, S, \mu)$  (10%)

二、(1)  $(X, S, \mu)$  is a measure space

(2)  $f: X \rightarrow \mathbb{R}$  is a measurable function

Show that  $|f|^p$  is measurable, for  $0 \leq p$  (10%)

三、(1) Fix  $a \in X \neq \phi$

(2)  $S = P(X)$

(3) Define  $\mu: S \rightarrow [0, \infty)$  by  $\mu(A) = \begin{cases} 0, & a \notin A \\ 1, & a \in A \end{cases}$

Show that  $(X, S, \mu)$  is a measure space (10%)

四、Let  $G$  be a nonabelian group, then the order  $\alpha(G) \geq n$ . Please find this  $n$  for us and state your reason. (10%)

五、Let  $(Q; +, \cdot)$  be an integral domain. Please find a field  $(F; +, \cdot)$  for which  $(Q; +, \cdot)$  can be imbedded in. (10%)

六、Evaluate the integral  $\int_0^{\pi/3} x \tan^2 x dx$ . (12%)

七、Compute the sum of all real numbers  $x$  such that

$$2x^6 - 3x^5 + 3x^4 + x^3 - 3x^2 + 3x - 1 = 0. \quad (12\%)$$

八、Let  $f$  be a nonnegative function which is integrable over a set  $E$ , show that given  $\varepsilon > 0$ , there is a  $\delta > 0$  such that for every set  $A \subset E$  with  $m(A) < \delta$  we have

$$\int_A f < \varepsilon. \quad (12\%)$$

九、(1) Show that if  $f$  is integrable over a set  $E$ , then so is  $|f|$ , and

$$\left| \int_E f \right| \leq \int_E |f|. \quad (8\%)$$

(2) Does the integrability of  $|f|$  imply that of  $f$  (6%)