

# 國立高雄師範大學九十三年度博士班招生考試試題

系所別：科學教育研究所

科目：分析與代數（全一頁）

1. (a) calculate : (5%)

$$\lim_{x \rightarrow \infty} \left(\cos \frac{1}{x}\right)^x$$

(b) Evaluate the double integral : (10%)

$$\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dydx}{x^2 + y^2 + 1} ,$$

2. Suppose that  $F(x)$  and  $G(x)$  are continuous when  $a \leq x \leq b$  and differentiable when  $a < x < b$ . Prove that there is a value  $c$  such that  $a < c < b$  and

$$\frac{F(b) - F(a)}{G(b) - G(a)} = \frac{F'(c)}{G'(c)} \quad (10\%)$$

3. If  $G$  is a finite group whose order is a prime number  $p$ , prove that  $G$  is a cyclic group. (10%)

4. In a commutative ring  $R$  with identity  $1_R \neq 0$ . Prove that  $P$  is prime if and only if the quotient ring  $R/P$  is an integral domain. (15%)

5. Let  $w = f(e^{x-3y}, \log(x^2 + 1), \sqrt{y^4 + 4})$ , find  $\partial_x w$  and  $\partial_y w$  in terms of the partial derivatives of  $f$ . (12%)

6. Suppose that  $f$  is continuous on  $[a, b]$ , and  $\phi$  is of class  $C^1$  and increasing on  $[a, b]$ . Show that there is a point  $c \in [a, b]$  such that

$$\int_a^b f(x)\phi(x)dx = \phi(a) \int_a^c f(x)dx + \phi(b) \int_c^b f(x)dx. \quad (12\%)$$

7. Show that there is no field of order 6. (12%)

8. It is impossible to trisect  $\pi/3$  by compass and straightedge. (14%)